

USING LESSON STUDY TO DEVELOP AN APPROACH TO PROBLEM SOLVING: ADDING AND SUBTRACTING FRACTIONS

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1. Introduction

We established the Practice Study Group of Arithmetic Education about 15 years ago in order to improve classroom instruction and to raise each other's abilities focusing on the young teachers in Sapporo City.

We have the Council of Sapporo Educational Research, a public research organization, and the Hokkaido Society of Arithmetical and Mathematical Education, a private research organization, which study arithmetic classes. However, many high ability teachers attend their meetings, making it difficult for young teachers to express their thoughts freely.

This limitation led to the proposal to form a study group where young teachers could participate actively. We thus established the Practice Study Group in Arithmetic Education.

This group meets once a month, and member teachers teach and observe about four open classes per year. Teachers of our group have visited the school and have observed each others' classes, and have spoken freely about the classes in order to raise the teacher's ability. For this class participation, we submit a request document for class visits to the principal of the school where a member teaches; thus far, principals have always granted permission..

In this paper we report on the effort to raise teacher's ability based on concrete practice examples (adding and subtracting fractions in the sixth), about the study method used, and describe the kind of practices and "good practice" for Teaching and Learning Mathematics through Lesson Study provided in our meeting for the studies.

2. Mathematics Education for Enhancing Student's Creativity: Instruction by Problem Solving Methods

Classroom instruction based on problem solving makes the most of activities by children based on their own initiative and judgment and emphasizes having children themselves finding solutions to problems.

Instruction in mathematics has tended to center on acquisition of knowledge and skills based on explanations by the teacher followed repeated drills. With "instruction centering on teacher-led explanations" and "instruction centering on drills" it is hard to get children enthusiastic about mathematics and to feel that it is really interesting. On the other hand, "instruction based on problem solving" aims not only to develop an "ability to think" and "ability to solve problems" but also to cultivate an "active attitude toward classroom learning" and an "ability to make active use of mathematics." at the goal is to get children to experience how much fun thinking can be, and through that, nurturing interest in and enthusiasm for an active attitude toward mathematics. Let us consider below why it is necessary to make such an improvement of shifting to instruction based on problem solving.

2.1. What is Beginning to Be Expected of Mathematics Teaching in Japan

A. Teaching the Basics

Efforts toward helping children acquire the basics of mathematics should be integrated with the aim of getting them to think on their own and express their own character and individuality. Furthermore, it is considered that acquiring the basics does not mean not only knowledge and skills but also includes abilities and attitude in learning content, the core of which requires thinking mathematically and problem solving. It is necessary, in order to carry out instruction based primarily on guidance for learning the basics, for the teacher to get as clear a grasp as possible of the content.

Let us divide the basics into two general aspects and explain each of them.

(1) The Content Aspect

One aspect of and the basics basics is the content. It includes the contents in the textbooks, which are generally considered the knowledge and skills that are divided into the instructional content for each grade. Examples are "addition up to 10," "multiplication up to 9 times 9," "calculation of fractions" and "measurement of angles" and the knowledge, understanding and expression and processing skills acquired through them.

Another part of the content aspect is thinking mathematically as the basis for producing knowledge and skills. It is the core of the content of learning the basics. It is necessary to foster an ability to understand content, appreciate its usefulness and learn to apply it to other things on the basis of the child's development up to the present grade in school through the content of the instruction in the different areas of mathematics. The following are examples of thinking mathematically:

- Expressing numbers in terms of place value, and thinking in terms of units, rate, ratios
- Thinking logically—drawing analogies, reasoning inductively and reasoning deductively

- Thinking in terms of functions and paying attention to constituent elements in figures

(2) The Method Aspect

The method aspect of the basics consist of problem solving and learning abilities. Although not all of the method aspect can be distinguished from the content aspect, it is a good idea to distinguish the following kinds of abilities and attitudes in instructional practice:

- Proceeding with classroom instruction on the basis of the children's own questions concerning what is being sought and how to find it
- Letting the children themselves form a general idea on how to solve the problem themselves, plan how to go about it, and then find the answer on their own
- Encouraging the children to utilize already acquired content and experience and develop it further
- Having the children take notes on the classroom proceedings to be used in group exchanges and self-evaluation
- Encouraging them to actively communicate with one another so as to learn from one another as a group

Interest, enthusiasm and attitude are important in terms of stimulating intellectual curiosity, thus serving as a driving force in getting children to willingly and actively come to grips with mathematics as an object of learning. These constitute a mental tendency regarding the different viewpoints of thinking mathematically—expressing, processing, knowing and understanding, which are necessary for developing students' problem solving ability.

B. Emphasis on Children's Own Initiative

There should be more emphasis placed on children's own initiative in classroom learning of mathematics. It is important that children discover the meaning of quantities and figures and come to have an awareness of mathematics and increase their depth of knowledge through experiences such as observation and experimentation and moving their bodies inside and outside the classroom.

The different ways individual children think should be given importance in instruction of mathematics. Furthermore, by sharing their ways of thinking, children are able to acquire more versatile viewpoints. In classroom instruction, deductive, inductive and analogical reasoning are frequently required of children. Also, in many cases they can solve new problems using knowledge and reasoning that they have already learned. What is being asserted here can be expected to contribute significantly to nurturing the basis for their creativity.

It is also important to nurture in children the attitude of making active use in their

everyday lives of what they learn about mathematics in the classroom. For that purpose, it is essential in teaching mathematics to relate it to everyday phenomena and to help children understand that everyday life contains lots of mathematical problems. One significant way of so doing is to encourage them to pose problems of their own using what they have learned in mathematics class. For instance, after they have learned the meaning of " $2 + 3$ " and how to calculate it in mathematics class in the first grade, the teacher can ask them to formulate problems concerning situations in which the answer can be obtained in terms of " $2 + 3$."

In order to attain this goal it is also important to provide them with training that makes it possible for them to express themselves in everyday situations using mathematical terms.

C. Emphasis on Enjoying Mathematics

Mathematics should be taught in such a way that children enjoy it and obtain satisfaction from it. The basis for making mathematics fun for children is to help them feel that they understand it, which will lead to the feeling that "thinking mathematically is fun." That being the case, the teacher has to show ingenuity in mathematics class from the viewpoint of showing how much fun and how interesting and worthwhile it is to learn mathematics and how wondrous it can be. If the children use the mathematics that they have learned to solve problems in various situations around them, they will learn to appreciate how much fun and how useful it is learning it.

It is also important to teach children through mathematical activities how much fun it is to learn mathematics. There should be many situations in mathematics in which children can experience a sense of discovery and even excitement and express it in words like "Of course!" and "Yeah, I see!" For that, children have to be encouraged to think for themselves. Just listening to the teacher's explanation and doing a lot of drills will not result in the feeling on the part of the children that mathematics is interesting and even fun, because they will often end up thinking that they "can't do it" or "don't understand" when they run up against more difficult problems.

There ought to be a lot of situations in mathematics class where children can encounter discovery, emotion and satisfaction of attainment. What it takes to make mathematics seem interesting and fun is to have them experience those feelings as often as possible. The more children come to like and enjoy mathematics through experiencing how interesting and even how much fun it can be, the better. We must not give up on children who have not been very good at mathematics so far. They, too, can learn to think "That mathematics class was interesting." We must not continue with teaching methods that produce feelings in children like "I don't want to do mathematics anymore!" and "Thank goodness there isn't mathematics anymore!" What we have to aim for is the kind of classroom instruction that can turn the consciousness of children concerning mathematics

in the direction of "mathematics is really interesting!"

What is required of school education is that it develops a firm rooting of the basics in children's minds and turn out children who are able to learn by themselves, think on their own, use what they have learned and show creativity inside and outside the classroom. Furthermore, the aim of the kind of mathematics instruction in the classroom described above is acquisition not just of knowledge and skills but also of capabilities and positive attitude regarding mathematical thinking, learning focused on problem solving, and so on. When engaging in instruction that intentionally puts the accent on acquiring the basics, it is necessary to have an attitude of instruction characterized by effort to grasp the content of the instruction as clearly as possible.

It is important that the children comprehend mathematics and that they develop the ability to apply the content and methods taught in mathematics class in order to solve problems that arise in their everyday lives. That goal cannot be attained with instruction in only one direction and with teaching that results in acquisition of what seems like knowledge and skills but really is not. If attention is paid to the children's process of thinking, and if they share their thinking with each other, they will be able to see things better and think better, and that tendency will spread. That is why "instruction based on problem solving" is considered to be the most appropriate method of teaching mathematics.

3. Concretization of Instruction by Problem Solving Methods

3.1 Instruction Based on Problem Solving

In order to build instruction based on problem solving, it is necessary to consider what makes instruction characterized by emphasis on acquisition of the basics. Such instruction goes beyond the basics and entails inclusion of the viewpoints of setting a clear image, sorting out the problems that have to be ironed out with regard to traditional instruction in the past, integrating such problems with improvement through shifting of the focus to problem solving and rethinking evaluation of learning. In that connection it is important to consider the following points:

- Awareness of the overall curriculum plans for mathematics
- Formulation of concrete instruction plans for the different units of instruction
- Definition of how the class hour of instruction is to proceed and how the situation regarding acquisition of and the basics is to be determined

The following points are also important in the case of instruction based on problem solving:

- (1) Achieving the result of initiative on the part of the children themselves in instruction based on problem solving.
- (2) Preparation of materials for the instruction that are suitable for the content to be taught and in tune with the needs and lives of the children.

- (3) Setting of instructional goals in tune with the actual conditions of the children and the educational tasks of the school and relating the difficulties of acquisition of the basics with the methods of evaluation of such instruction.
- (4) Supporting activities that stimulate the enthusiasm and problem awareness of the individual children and that encourage them to think and pursue solutions on their own.
- (5) For group discussion activity that can lead to better problem solving, changing from the kind tailored to the teacher to the kind based on the viewpoint of the children themselves that can serve as a forum for discussion and communication in which they themselves share their values.

In general, instruction processes such as those indicated below (underlined) come to mind regarding classroom instruction based on problem solving, the aim in each process (step) being acquisition of ability and the necessary attitude concerning problem solving.

- Formulating the problem
- Understanding the problem
- Planning solution of the problem
- Carrying out the solution
- Consideration of the solution

Instruction processes A to E provide further elaboration of the above points:

- A. Understanding and grasping the meaning of the problem (collecting and sorting out information constituting the problem and formulating the problem oneself, getting familiar with the problem situation regarding the given problem and conceiving it as one's own problem)
- B. Planning solution of the problem (preparing the conditions and information needed for solution, already acquired experience and knowledge, skills, ways of thinking, etc.) and getting a rough idea about how to go about finding the solution)
- C. Carrying out problem solving (reaching a tentative conclusion concerning the content of the solution (formation of concepts, acquisition of knowledge and skills, becoming aware of mathematical ways of thinking, etc.) through trial and error in mathematical activities)
- D. Consideration of the solution (collate and check the results with what was expected; the children consider the different contents of each other's solutions as a group and arrive at a more refined solution)
- E. Final summing up and looking back on the processes of solving the problem (confirmation of the state of attainment of the goal (things like state of acquisition of the basics and realization of the evaluation criteria) as a basis for students' own evaluation of classroom activities)

Classroom instruction based on problem solving is a method of instruction that emphasizes the children's activity based on their own judgment and the process of solution by the children themselves in working toward the goal of the instruction. It is

therefore important that the teacher presents problems suited to that goal and works to support the children's own independent activity. Particularly important are the questions that the teacher poses during the class. In the guidebook the main problem to be dealt with in the class period is put in a rectangular box, and the questions posed by the teacher are written in gothic script.

3.2 The Significance of Question Posing in Classroom Instruction Based on Problem Solving

During classroom instruction the teacher is expected to talk to the children in such a way as to get them to better manifest their thinking and behavior, exploring their individual inner minds and understanding their individual characters and personalities. Let us define such "putting questions to and spurring" the children individually and as a group by the teacher in agreement with such a desirable picture of classroom instruction based on problem solving as "question posing."

Ingenuity and improvement in question posing should not be just for the smooth progress of instruction by the teacher but rather for the purpose of achieving improvement in inadequate points and points in which sufficient results are not obtained in as-is classroom instruction, focusing on "how to get the children to make progress" by proceeding with classroom instruction on the basis of problem solving.

Question posing should not be a one-sided affair, but rather aimed at getting the children to react and respond; the point of ingenuity here is to lead to "dialogue" both between the teacher and the children and among the children themselves, which is essential to establish communication in the classroom.

Question posing is considered to be the function of eliciting and assisting the children's thoughts in connection with acquisition of knowledge and skills and formation of mathematical way of thinking and necessary attitude. In eliciting the thoughts of individual children, one should not expect them to be appropriate and valid as a complete whole, and the direct purpose should not be that of having them announced to the whole class as such, but rather the basic aim should be that of simulating the children's inner minds and thought processes.

Response is elicited by stimulus. But priority should not be given to getting response for the sake of convenience of the teacher in his or her instruction. Rather, the main point should be using response for promotion of the child's thinking activity and getting the children to talk with one another about their thoughts for deeper appreciation of each other's thinking

3.3 Example of Question Posing in Classroom Instruction Based on Problem Solving

Teaching Plan of a Mathematics Lesson

Students: Sixth grade, Elementary School in Sapporo

20 boys and 17 girls, total 37 pupils

Teacher: Masu Kanno

1. Unit : Adding and subtracting fractions

2. Aims and the flow of learning fractions

(Aims)

- Interests, attitudes, motivation

To understand the situation where adding and subtracting fractions with unlike denominators are used and willingly try to solve the problem with the knowledge already acquired.

- Mathematical thinking

To understand it is possible to solve the problem by using diagrams or making the denominator, which is the measuring unit of quantity, the same number. Then, to think up reducing fractions to a common denominator as the way of calculation.

- Expression, skill

To be able to simplify fractions and to be able to convert the fractions to same denominator.

To be able to calculate addition and subtraction of fractions.

- Understanding, knowledge

To understand the meaning and the method of simplifying fractions and converting fractions with unlike denominators to fractions with like denominators.

(The flow of learning fractions)

Fourth grade	<ul style="list-style-type: none">· The meaning and the notational system of fractions
Fifth grade	<ul style="list-style-type: none">· Adding and subtracting fractions with the same denominator· Equivalent fractions—simple case· Writing the answers of dividing whole numbers in fractions· Relating fractions to decimals, relating decimals to fractions
Sixth grade	<ul style="list-style-type: none">· Equivalent fractions, how to make equivalent fractions· The meaning of simplifying fractions and reducing fractions to a common denominator· Adding and subtracting fractions with different denominators· The meaning and calculation of multiplying fractions· The meaning and calculation of dividing fractions

3. About This Teaching Material

(1) The value of this teaching material

The main aim of the sixth grade lesson “Adding and subtracting fractions” is to deepen the student’s understanding of the meaning of fractions and to develop their ability to calculate with fractions.

The concrete teaching items are

- a. to understand that the fractions made by multiplying the same number by the numerator and the denominator does not change the value.
- b. to put together how to check equivalent fractions and how to compare fractions
- c. to be able to calculate adding and subtracting fractions with different denominators.

In the fourth grade lesson on “fraction”, the students have learned the meaning and the way to write fractions. In some simple cases they have learned there are equivalent fractions. In addition, they have learned adding and subtracting fractions with the same denominator and expanded their view of numbers and calculations.

Also, they have studied adding, subtracting, multiplying and dividing whole numbers and decimals. Whole numbers and decimals are written in the decimal system, and they have deepened their understanding and skills of the four rules of arithmetic within the system.

In this teaching unit, they study adding and subtracting fractions, which are not written in the decimal system. Accordingly, we have to help them understand the meaning of fractions through many different situations. In order to do that, at the introduction of this unit, I let them pose questions to link the meaning of a concrete situation and the adding and subtracting of fractions. Also, by using these questions throughout the unit, they will be able to have the perspective of the whole unit. Furthermore, as the study goes with their questions, we can expect their enthusiastic attitudes.

The meanings of “reducing fractions to common denominators”, “simplifying fractions” and the method of adding and subtracting fractions tend to be taught in a mechanical way. However, we would like to make the most of students’ ideas and organize a lesson as if they find things by themselves and feel the merit of using the idea of common multiples in fractional calculations.

(2) The ability we want to cultivate in a student

(With reference to the content)

- a. Adding and subtracting fractions become possible by reducing the fractions to a common denominator.
- b. The adding of mixed fractions is the adding of "whole number + proper fractions” based on the idea of a measuring unit.
- c. To understand that there are many ways to write the same equivalent fraction by using diagrams.

(With reference to the aims)

a. Mathematical thinking

To help them find the rules to make equivalent fractions.

$$\frac{a}{b} = \frac{a \times c}{b \times c} (c \neq 0), \frac{a}{b} = \frac{a \div c}{b \div c} (c \neq 0)$$

b. Logical thinking

There are many equivalent fractions. Simplifying a fraction means to express it in the simplest form, i.e., to express it by using the smallest denominator. Reducing fractions to a common denominator means to express each fraction by using the same denominator.

In adding and subtracting fractions, it is important to express the fractions by using the same denominator (that is, the unit of measuring) and think in an orderly fashion.

c. Generalization

Through the learning of adding and subtracting fractions with unlike denominators, the students learn the points in common and the points of difference in calculating whole numbers and decimals. They pay attention to correlations among the groups of numbers.

d. Estimation

i) The prospect of a unit:

In this unit, students make up problems in the first class of mathematics with the aim of understanding the case of adding and subtracting with different denominators. The whole unit consists of using these problems made by students, thereby, it seems to the children that they can foresee the contents of study of the whole unit.

ii) The insight of reducing fractions to a common denominator:

In this lesson, we take the subject of (unit fraction)-(unit fraction), and want children to discover the necessity of changing fractions to common denominator or reduction of a fraction.

We have observed that students can find the fraction as a unit by themselves if it is introduced as subtraction rather than addition. Moreover, I want to estimate a solution at the time of introduction. This activity will also help students to find the fraction used as a unit.

Following these two classes, the next two classes will study reducing a fraction and changing fractions to a common denominator. Then we will spend 4 classes on various fractions with different denominators. In these classes, students will study how to add and subtract these unit fractions with different denominators by themselves.

iii) Estimation of solution:

Students will estimate a solution, paying attention to [merit of changing fractions to a common denominator] = [the merit of a common multiple]. We think students can realize

by themselves the merit of estimation in the concluding 10 classes.

iv) The prospect of domain:

For studying the domain of “numbers and calculations”, usually the following order is taken: “Understanding of a phenomenon” → “formula representation” → “study of algorithm” → “application”. Thus, if the order of progressing study is known, when students advance in their study, it will be effective.

The above will be the basis for the following study of the domain of “numbers and calculations”, when students understand the merit of learning in such an order from the studies in this unit. For this reason, it is useful to review the whole unit at the end of the unit.

4. Teaching Plan (13 hours)

1 st	Let's make problems of adding and subtracting fractions.
2 nd / 3 rd	(See 5.)
4 th	Is it possible to subtract fractions if the denominators are the same? It is possible to subtract fractions with like denominators. To find common denominator, it is easy and fast if we use common multiples.
5 th	Do the fractions $\frac{1}{6}, \frac{2}{12}, \frac{3}{18} \dots$ have different values? The values of fractions are equal if both the denominator and numerator of a fraction are multiplied or divided by the same number.
6~9 th	1. Adding proper fractions (No carry up) 2. Adding proper fractions (Carry up, simplify fractions) 3. Adding mixed fractions (Carry up, simplify fractions) 4. Subtracting proper fractions from mixed fractions (Carry down) 5. Subtracting mixed fractions from mixed fractions (Carry down) 6. Addition and subtraction of three fractions
10 th	Discussion
11 th	Let's express time in fraction
12~13 th	Practice: Encourage each students to learn

5. Detailed Plan of the Second and Third Classes (Appendix I)

Aim:

To find the common unit of measure during the activity of comparing quantities.

To notice that subtracting fractions with unlike denominators is possible if the fractions are expressed with the same denominator.

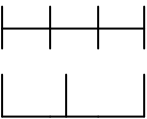


To express quantity by using diagrams or equivalent fractions. To try to calculate

the difference between them.

6. The device of question posing in this class

Question Posing	Content (corresponding thinking)
1) (After getting a formula $1/2 - 1/3$) How can you estimate the solution? (Nice question: Is the answer close to 1?)	From the information acquired from the problem sentence or formula, students estimate a quantity of the answer (difference) in question.
2) Which part of the area figure is asked in this problem?	In the area figure which the child showed, it clarifies which portion corresponds to the answer (difference) of this problem. (The clarification in question)
3) Why can J-kun understand this part is $1/6$?	The basis and reason of the idea are clarified for how the student considered and found the difference of one half and $1/3$ was $1/6$ (Reason and deduction)
4) How did K-kun consider it was how many parts of the remaining part?	To find the difference of $1/2$ and $1/3$, they compare the fractions to 1 and find a common unit to arrive at equivalent fractions. (Clarification of a thought)
5) Today, we learned subtraction of fractions. How can you calculate it?	Promoting the rearranging of learning of today's class, students notice they able to calculate subtraction of fractions if they convert the fractions to the same denominator. (Generalization)

Appendix I

Learning Activity	Student's thoughts	Teacher's Activity
<p>Which of $\frac{1}{2}l$ milk and $\frac{1}{3}l$ juice contains more and how much is the difference? ?</p> <p>Expression: $\frac{1}{2} - \frac{1}{3}$</p> <p>a. $\frac{1}{2} - \frac{1}{3} = \frac{1-1}{2-3}$</p> <p>b. compare the difference of $\frac{1}{2}$ and $\frac{1}{3}$ with the whole (Numerical line) (Area diagram)</p>   <p>(Tape diagram)</p>  <p>c. Compare the difference by finding the common unit. (Tape diagram) (Area diagram)</p> <p>(Common multiples)</p> <p>It is possible to subtracting fractions with unlike denominators if we write the fractions by using the same denominators.</p> <p>Is it possible to calculate other subtracting problems of fractions?</p> <p>How can we find the fractions with same value?</p> <p>We got many answers. Do they have different values or not?</p>	<p>Probably the answer is smaller than $\frac{1}{2}$.</p> <p>I can't subtracting because the denominators are different.</p> <p>If they have the same denominators, it's possible... I'll compare them by drawing diagrams.</p> <p>I know which contains more, but I don't know the difference.</p> <p>Let's compare the difference with the whole.</p> <p>The amount of $\frac{1}{2}$ and $\frac{1}{3}$ are $\frac{3}{6}$ and $\frac{2}{6}$ if the nicks are changed. Then the difference is $\frac{1}{6}$.</p> <p>We had learned that $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$ in the fourth grade. That means the difference is $\frac{1}{6}$.</p> <p>It is possible to subtracting if the denominators are the same.</p> <p>Is it possible to subtracting fractions in other cases?</p> <p>There are many fractions which have the same value.</p> <p>How can we find them?</p>	<p>Let the students imagine and understand the situation of the problem.</p> <p>Encourage them to estimate the difference. Encourage the students who can't have the perspective to think in the diagram or remember what they had learned before.</p> <p>Teach each student while walking around the classroom.</p> <p>Let the students pay attention to the relation of the diagrams and the numerical expression.</p> <p>Let them consider in other problems.</p> <p>Let them consider what they want to learn next.</p>