Refining Communication to Improve Mathematics Didactics: A Case Study

Ui Hock CHEAH
Southeast Asia Ministers of Education Organization
Regional Centre for Education in Science and Mathematics
Penang, Malaysia

Abstract

This paper presents a preliminary report of a case study on a lesson conducted as part of a project to introduce the Lesson Study approach to a group of teachers. The lesson which was planned collaboratively by three teachers to focus on mathematical communication and thinking was then carried out by one of the teachers. The lesson was videotaped and the communication that took place was then transcribed and interpretively analysed. The findings of the study revealed that the lesson tasks designed by the teachers were generally able to stimulate active pupil participation in the lesson. However the communication in the lesson was mostly focused on the teacher attempting to lead the pupils to arrive at his answers. While the study did raise some issues as to the way mathematical communication is carried out in the Malaysian primary classroom, it was found that the Lesson Study method was a suitable and non-threatening approach for teachers to improve and further develop mathematical communication in the classroom

Introduction

Communication in the Malaysian Mathematics Curriculum

Communication in the Malaysian mathematics curriculum is conceptualised in the context that instruction should collectively highlight the processes of communication, problem solving, reasoning, and making mathematical connections. These processes are seen to be essential in the classroom that focuses on mathematical thinking (Lakatos, 1976; Doerr, 2006). In clarifying ways of enhancing mathematical communications, the curriculum highlights three main areas of communication: values and aims of communication, oral communication and written communication (Curriculum Development Centre, 2006).

Values and aims of communication. Several considerations were suggested which includes identifying relevant contexts, pupils' interest and teaching materials, ensuring active, stimulating meta-cognitive skills, inculcating positive attitudes and creating a conducive learning environment.

Oral communication. Some of the suggested communication techniques include story-telling, asking and answering questions, structured and unstructured interviews, discussions and presentation of assignments.

Written communication. The curriculum suggests communication activities such as doing exercises, keeping scrap books, keeping folios, undertaking projects and doing written tests.

Essentially the Malaysian curriculum places mathematical communication within the context of a conducive learning environment where mathematical thinking is emphasised. In contrast to the more traditional didactics where mathematics is developed through mainly deductive methods, the more progressive approaches now emphasise argumentation and reasoning to facilitate the construction of mathematical ideas. Lakatos (1976) argued that heuristics and processes such as conjecturing, critiquing and providing counterexamples are important processes in mathematical problem solving. Mindful that the Malaysian perception of school is much alike that of the East Asian perspective that places emphasis on product rather that processes, and on effort in achieving success in doing mathematics (Leung, 2000; Lim, Fatimah & Tan, 2003), efforts to focus on mathematical processes would need to consider changing the mindset of the teachers. Teachers need to be convinced that focusing on mathematics processes will actually be a better alternative in producing mathematical success especially in student achievement. Thus an equally important consideration is the need for teacher support to assist teachers make the mental shift towards mathematical thinking in the classroom.

Mason, Burton and Stacey (1982) examined the different phases of problem solving and suggested some heuristics to assist the learner in the problem solving process. Some cues that were suggested to assist the learner in problem solving were to clarify what the pupil already knows and what he needs to know, conjecturing, justifying and convincing, and specializing and generalizing. The role of the teacher is thus to probe and ask relevant questions in order to assist the pupil move towards solving the problem (Richards, 1991). Much of the suggestions can be seen taking place in the Lesson Study videos on Japanese classrooms (e.g. Hosomizu, 2006). In the videos, the lesson starts with a rich mathematical task where the pupils work together to arrive at mathematical ideas and formulas. First the teacher probes the students understanding of the problem; what the students already know, and what the students want to know in the problem. The teacher then encourages the pupils to suggest solutions and make conjectures. He probes the pupils' thinking and thoughts, using questions to cleverly invoke the pupils' thinking until they arrive at the solution which is acceptable by both the teacher and other peers (Simon, 1995; Steffe & D' Ambriosio, 1995).

A framework for communications that is suggested in this paper places communications in the context of the classroom together with other important criteria for planning a good lesson: 1) rich tasks which enable the pupils to engage in mathematical thinking, 2) constant evaluation of the lesson by the teacher both during and after the lesson and 3) the creating of a suitable environment so the mathematical discourse can take place (National Council of Teachers of Mathematics, 1991; Bahagian Pendidikan Guru, 1998). Figure 1 shows the four important aspects of a lesson focusing mathematical thinking and the key features of classroom communication (Cheah, 2007).

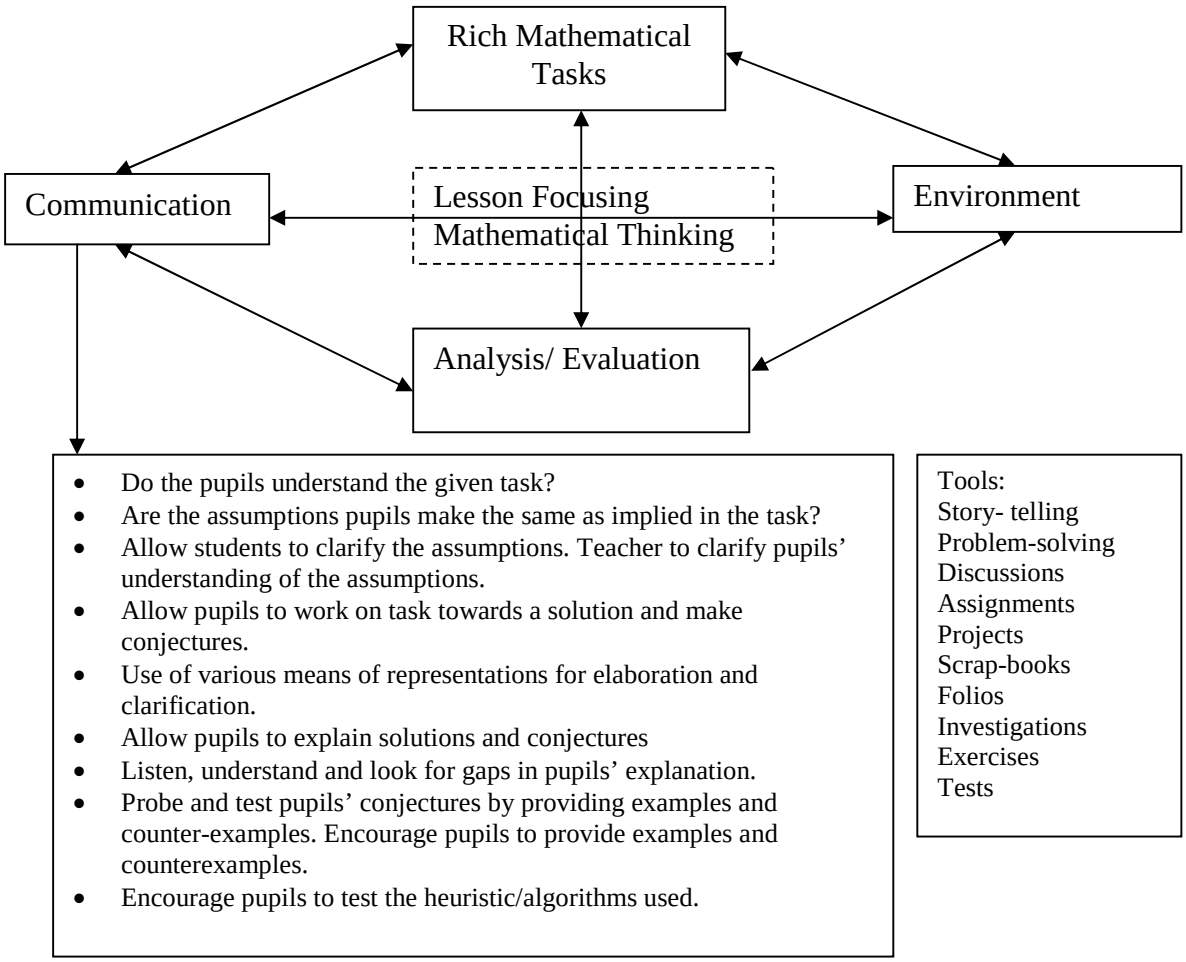

Figure 1. A framework for communications in the Malaysian primary mathematics classroom.

Components of Mathematical Communication in the Primary Classroom

An important aspect of communication in the mathematics classroom is that it involves the values of the community. In the case of the primary classroom, the teacher who is obviously more mature and knowledgeable than the children holds the key in value transmission in the mathematics classroom. Not only do the pupils learn mathematical facts and ideas but they also acquire the informal knowledge about mathematics: what it means to study and do mathematics and the values the nature of the subject carries through the informal activities of the lesson such as through communication (Bishop, 1988). Keeping in mind the constraints and impediments in attempting to carry out a more process-focused mathematics lesson, the following features are suggested as appropriate components of communication in the primary mathematics curriculum in Malaysia:

1. Classroom communication is very much influenced by the multilingual and multicultural nature of the classroom. While communication on a formal level in Malaysia is carried out using English, it is often the second language for both the teachers and pupils. Granted that there is a growing urban middle-class that

speaks English at home, it is however generally an uncommon occurrence in Malaysia. This scenario creates constraints and challenges for both the teacher and students as they engage in mathematical discourse (Clarkson, 2007).

2. Communication can be enhanced through various means of representation, such as through symbols, diagrams, drawings, charts and graphs which are commonly used in mathematics. This feature becomes an even more important consideration when mathematical communication is carried out in a second language.

3. Communication is dialectic and to encourage its use in the classroom, teachers need to develop a belief that even when they are in authority; there is a need to empower the pupils and coax them into dialogue. This would mean that teachers should patiently probe pupils' thinking through questioning and allowing and encouraging pupils to elaborate on their ideas instead of just telling.

4. Communication in the mathematics classroom ought to promote the value that mathematics is rationale. What is acceptable as right in the classroom is not through authority but rather through mathematical reasoning and logical arguments?

5. Communication is used as a means of promoting the idea that mathematical knowledge is developed through collaboration and not by authority.

Methodology

The purpose of this case study which adopted an interpretive approach was to examine and suggest ways to further improve mathematical discourse in the Malaysian primary classroom. It formed part of a larger study aimed at introducing Lesson Study to a group of 20 primary school teachers. The main source of data for this study was the study lesson which was videotaped and transcribed.

Context of the Study

The Participants

The three teachers in this study, Samy, Kavitha and Anita (not their real names), were among twenty other teachers who attended a series of five three-hour workshops which were held with the aim of introducing the ideas of mathematical thinking and communication and the Lesson Study approach. All the teachers who attended the workshop were recommended by the head teacher of the respective schools and volunteered to take part in the study. Samy was a young teacher with three years of teaching experience. Anita had ten years of teaching experience and Kavitha was a senior English language teacher. Both Samy and Anita taught mathematics in the school.

The Workshops

The workshops which were attended by the teachers were designed with the aim of introducing the ideas of mathematical thinking and communication and Lesson Study and to assist the teachers incorporate these ideas into their mathematics lesson. During the first workshop the teachers were introduced to the idea of Lesson Study as a school-

based approach in teacher development. The second workshop was focused on having the teachers work on mathematical thinking tasks themselves and highlighting the importance of communication when they were working on the task. The third workshop focused on the Lesson Study concept and the formation of Lesson Study group. During the fourth and fifth workshop, the teachers broke into groups and began to discuss and plan the study lesson plans together.

The School

The school where the study lesson took place was an urban vernacular co-educational primary school with a population of about 300 students. The main medium of instruction was Tamil. However mathematics was taught in English, in line with the English in Science and Mathematics policy currently practiced in Malaysia. The pupils in the school came mostly from working class homes and about fifteen per cent of the pupils were from the orphanage which was located next to the school. Historically the school had served as the feeder school for the children from the orphanage.

The Study Lesson

The study lesson was planned collaboratively by Samy, Kavitha and Anita. The lesson plan was discussed and revised twice with the collaborative assistance of the researcher. The main reason for the revisions was that through discussions the teachers realized that the initial tasks that they designed were not suitable to invoke thinking and communication. The planned lesson was then carried out by one of the teachers. Samy taught the lesson while the other teachers together with the researcher observed the lesson and took notes.

The Lesson

There were 25 eight-year-old children in the class and the lesson was focused on the topic of shapes and space. The main aim of the 60-minute lesson was to allow the pupils to compare and sort triangles, squares, rectangles and circles according to their properties.

The Tasks

There were four tasks in the main lesson development together with the introduction and the closure (See appendix A). In the introduction the pupils were required to identify the different shapes (squares, rectangles, triangles, circles) that were used to make a picture of a house. The first and second tasks in the lesson development focused on the pupils grouping shapes into categories and explaining their properties. The third task in the lesson development was aimed at the children working together to find out the difference between rectangles and squares. Each group of pupils was given two squares of the same size and two rectangles of the same size. The pupils were then asked to find the number of possible ways to fit one square on top of the other square by rotating it. Similarly the pupils were asked to fit the rectangles by rotating one rectangle on top of the other. In the fourth task in the lesson development, the pupils were required to complete worksheets.

Findings and Discussion

The Mathematics Discourse

The analysis of the video and the teachers' comments after the lesson shows that generally the pupils participated actively in all the lesson tasks.

Further analysis of the video revealed that the total talk time constituted approximately about half of the lesson time. The rest of the time the pupils were doing seatwork. Out of the total talk time, teacher talk time was about five times more than that of pupil talk.

On many occasions teacher talk was focused on questions that closed and were aimed at the pupils attempting to arrive at answers that the teacher wanted. For example the teacher-led communication was observed in the following excerpt transcribed from the set induction task.

(L19) Teacher : Ok, this roof is made of…

(L20) Student : triangle

(L21) Teacher : this door?

(L22) Student : rectangle

(L23) Teacher : this window?

(L24) Student : square

(L25) Teacher : this chimney?

(L26) Student : triangle

(L27) Teacher : this one long rectangle and triangle, right ok.
Today's lesson we learn about shape. First one its…

(L28) Student : square, rectangle, triangle

The third task in the lesson development required the pupils to inquire into the difference between a square and a rectangle. To do this the pupils were given two squares of the same size and were asked how many times they could fit one square on top of the other through different rotations of the square on the top.

(L75) Teacher : Now we're looking for the square
Now this turn for this group.

(L76) Teacher : Now we're looking for this square
How many times you can paste it

(L77) Student : Four

(L78) Teacher : Just show it

(L79) Teacher : This first, second, third, and then four…
For the square you can twist four sides, right! First, second, third, fourth. all four side ok.

(L80) Student : one

(L81) Teacher : Rectangle have….
have two opposite side
For the square…..

(L82) Student : same side

(L83) Teacher : this is the different for the square the have same size. because square have the same side. That rectangle why? because only has two opposite same side. That the reason… Understand class

While the task was designed to invoke pupils' thinking, it was found that the teacher did not use the opportunity to ask the pupils to reason why a square is different from a rectangle as seen in the transcribed conversation (L75 to L83). Instead the discourse was focused at attempting to have the pupils arrive at the reasoning predetermined by the teacher. There was a lack of opportunities for the pupils to reason and show that the reason for being able to rotate four times and fit the square each time was due to the fact that all four sides were the same. The teacher seem to be satisfied when the pupils were able to say that the square has four equal sides and that the rectangle has two opposites sides that are equal. It was however observed that earlier while the pupils were working on the same task the pupils were able to reason why a particular rotation of the rectangle does not fit the other rectangle.

(L59) Teacher : how many times you can paste on it.

(L60) Teacher : Can?

(L61) Student : no

(L62) Teacher : can?

(L63) Student : ya

(L64) Teacher : This is cannot why

(L65) Student : Because that have blank side in there

During the third task of the lesson development, the teacher asked the class how they could make a triangle from a square.

(L91) Teacher : from this square you can make two triangle

(L92) Student : triangle have four corner.

(L93) Teacher : triangle have four corner?

(L94) Student : not

(L95) Teacher : triangle have four corner?

(L96) Student : not

(L97) Teacher : from this paper how you can make triangle.

(L98) Student : the triangle has three corner, has three side.

The pupil's answer in (L92) shows that she was arrived at the inference that the triangle has four corners. The teacher missed the opportunity to assist the pupil to think futher and explain why she said that a triangle has four corners. Instead the teacher's repeated questioning of the pupil's answer (L93 and L95) which led the pupil to change her answer and say that the triangle has three corners and three sides. The teacher could have explored the reasons for the pupil's comment that the triangle has four sides (L92) in order to further understand the pupil's initial conception first before assisting the pupil to reconstruct her initial mathematical concepts about the properties of a triangle.

One observation from the lesson analysis concerns the use of English as the medium of the discourse. It was observed that while both pupils and student did use English during the lesson, the pupils could be heard using the Tamil language among themselves while working in groups on the tasks. Another observation was that the use of English was not accurate at times. For example both teacher and pupils would say, "The rectangle has two opposite sides. The square has four sides." From the point of view of mathematics content, it is important that pupils should acquire exactness of mathematical language. It is through this preciseness that a good foundation of mathematics is built.

The Teachers' Views of Lesson Study

Interviews with all the three teachers revealed that they found the Lesson Study approach as a suitable way to improve the mathematics lessons. The following were the teachers' comments.

> "From this lesson study I can learn more especially when we are doing group work, we can discuss a lot of things, so I can improve myself. When in my normal lesson I just focus on my paperwork but for this lesson study I can get the students to think aloud and with a lot of activities like the group activities. From the group activities the students were able to discuss among themselves. I think this is one of the good opportunities to improve ourselves." (Samy comments)

> "When we first attended this program, I did not know what it was all about, gradually as we went through each session we gained an insight to Lesson study. This is an ongoing process which I feel helps the teachers to plan out a lesson effectively and to improve our teaching strategies. It also acts as a guide to prepare other lessons because we don't … It helps the teachers improve their lesson plan. And it also helps them prepare a lesson which is more creative. This should be a continuous process whereby all mathematics teachers should be involve to have… to conduct a good lesson, an effective lesson and to improve the speaking skills of the children to be more pupil oriented and teacher guided." (Kavitha's comments)

> "First three of us, we decide the activity for the lesson plan and then we do the lesson plan for one hour and then we change the lesson because there were no mathematical thinking and then we add the mathematical thinking activities in that lesson and then we run this lesson in my school. Pupils involve… active in all the activities. Pupils could answer teacher's question. Pupils also enjoy because they got many colourful teaching aids. I think lesson plan like this to help teacher to improve their teaching method and then this lesson plan can help other teacher to improve their teaching." (Anita's comments)

Limitations of the Study

Mindful that this is a preliminary report, several limitations of the study are noted. A richer analysis of the data can be achieved if the study also includes the teacher's views of the several episodes of communication reported in the finding. This could possibly lead to further examining the teacher's beliefs about mathematical communication. Further the data of the study was sourced from a single lesson conducted by the teacher.

Analysis of a few more lessons may be able to further increase the understanding of the teacher's actions in the classroom.

Conclusion

Communication is an essential part of the mathematical classroom. Students may use verbal language to communicate their thoughts, extend thinking, and understand mathematical concepts. They may also use written language to explain, reason, and process their thinking of mathematical ideas. Mathematical ideas are facilitated, constructed and internalized through discourse. This case study raises several issues in examining mathematical communication. In this study it is seen that the lesson tasks were designed to invoke mathematical thinking. It was observed that the tasks allowed the pupils to be actively involved in the process of constructing mathematical concepts but there was a lack of an attempt by the teacher to probe into the pupils' thinking. Most of the time, it was observed that the teacher would be directing the pupils to arrive at his answers. If the teacher had used questions that seek to understand the pupils' conception, he would have been able devise further pedagogical activities to assist the pupils construct their mathematical ideas. While the lesson tasks were designed so that the lesson could be pupil-centred, the communication in the lesson was most teacher-centred. There could perhaps be two reasons that can be explored for future research. The first is the teacher's conception of the purpose of discourse in the mathematics classroom. Essentially this means exploring the teacher's beliefs about the role of discourse in the construction of mathematical concepts. The second reason could be due to the lack of experience of the teacher who was just into his third year of teaching.

The lack of preciseness of the mathematical language used by both the teacher and pupils is also a concern since preciseness is an important feature in the construction of mathematical ideas. This raises the issue as to whether this lack of preciseness in the teacher's mathematical language could be due to the use of English as a second language to teach mathematics. Did the use of English create impediments so as to affect the precision of the mathematical language? Another possible reason for this observation could be the lack of depth of the beginning or novice teacher's content knowledge and pedagogical content knowledge. Both these issues can be further explored in future research

This study has revealed that discourse in the Malaysian mathematics classroom needs to be further studied. This is essential in order to further understand how teacher-pupil discourse can be effectively utilized to assist pupils' construction of mathematical ideas. However, the teachers' openness to the use of Lesson Study approach indicates the utility and suitability of it being used to further improve mathematics lessons both as a teacher development program and also as a research method.

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