DESIGN EXPERIMENT FOR TEACHING PROPORTION BASED ON CULTURAL-HISTORICAL ACTIVITY THEORY: PROCESS OF SYMBOLIZING THROUGH COLLECTIVE DISCOURSE

Minoru OHTANI

Institute of Human and Social Science, Kanazawa University

The purpose of this study is to design teaching unit of proportion in a sixth-grade Japanese mathematics classroom which has theoretical underpinnings of the Cultural-Historical Activity Theory. The following discussion consists of two parts. First part involves an indication of the theoretical framework and the justification for the methodology used. In the framework, cultural tools such as numerical table, graph and formula become symbol of proportion in which collective discourse plays supportive role. The second part involves description of hypothetical trajectory of appropriation of cultural tools in which table and line graph mediates interpersonal functions then become intrapersonal symbols of proportion through collective discourse. Data from actual teaching experiment buck up for legitimacy of the design.

INTRODUCTION

The purpose of this study is to design a unit for teaching proportion in a sixth-grade Japanese elementary mathematics classroom which has theoretical underpinnings of the "Cultural-Historical Activity Theory" (Leont'ev, 1975; Vygotsikii, 1984). The following discussion consists of two parts. First part involves a discussion of the theoretical framework for analysizg mathematical activity. In the framework, we will coordinate a theory of mathematical activity in the "Realistic Mathematics Education (RME)" (Gravemrijer et al., 2000) with sociocultural activity theory. There we will incorporate the concept of "cultural tools" (Vygotsikii, 1984) and "discourse" (O'Connor & Michaels, 1996) into RME theory in order for designing teaching unit of proportion. The second part involves description of hypothetical learning teaching trajectory (van den Heuvel-Panhuizen, 2001) where cultural tools such as numerical tables, graph and algebraic formula become symbol of proportion in which collective discourse plays supportive and generalizing role. Then data of teaching experiment are presented and analyzed in the light of the framework to generate a description of the process of symbolizing of numerical table. Results from the interpretation of the data reveal that the process of symbolizing consists of four phases in which cultural tools change its function form intermental to intramental one through collective discourse.

THEORETICAL FRAMEWORK

Mathematical activity in RME and Possible Coordination with Activity Theory

As mathematical activity, this paper verifies a discussion in Freudenthal Institute. Freudenthal (1991) considered an activity that organizes a phenomenon by abstract mathematical means and named it "mathematization". Treffers (1987) categorized mathematization into two types (horizontal and vertical) and logically typified mathematical education into four types according to with/without matthematization. The institute recommends the perspective that has both two types of mathematization and calls it “Realistic Mathematical Education”. RME theory is build on the basis of “levels of thinking”(van Hiele, 1986) which describe development of thinking during long-term span. In the early stage of RME, transition to a higher level of thinking was made by establishing a micro level of “progressive mathematizing” (Treffers, 1987: 247). After that, RME presented a new development which has three points (Gravemrijer et al., 2000). First is to set up four levels for move up the level of thinking. These levels are: a situation that has a sense of reality (level 1); a model construction underlying the pupils’ informal procedures (level 2); the model itself becomes targeted and tools for inference (level 3); and formal mathematical knowledge (level 4). Second is to expect the process that pupils make the model develop by themselves from level 2 to level 3. Third is to focus on symbolizing and communication, and think them as a vehicles to construct a formal knowledge.

The RME’s theoretical standpoint supporting a contemporary development is a “social constructivism” (Gravemrijer et al., 2000). This standpoint combines a sociological perspective that analyzes practice at classroom level with a psychological perspective that analyzes action at an individual level. This standpoint has, however, a criticism that the theory can be said as interdisciplinary but cannot be said as consistent. The major problem is that we only return the mathematical activity in the classroom to the two elements of sociology and psychology, but we do not mention the link between them (Waschescio, 1998). In fact, RME “describes” activity but does not “explain” how a personal informal knowledge combines with a formal knowledge, or how symbolizing and communication play a role then. As compensation to it, cultural-historical theory tries to explain the link between social practice and individual action. Originally, RME took a cultural-historical approach. Actually, the van Hiele (1986) emphasized role of language and guided orientation for explicating the structure. Treffers suggested that “cultural amplifier” (Treffers, 1987: 251) such as schemas, models, and symbols should positively be offered in order to consciously aiming at higher levels of thought. Thus, the problem of RME is not in itself, but it is assumed to be caused by excessive devotion to social constructivism. Therefore, reviewing symbolization and discourse from the cultural-historical theory will be our theme.

Cultural-Historical Perspective on Activity

The “Cultural-Historical Theory” today is called as “Activity Theory” (Leont’ev, 1975) and is useful to consider character of mathematical activities. There are three characters of activities in the theory. Firstly, when the word “activity” is used, it means a qualitative aspect. Namely, it is not quantitative strength, but is quality, especially “motive” is an index of activity. Secondly, activity is a cultural practice. Expressly, unique cultural tools are used in the practice. Thirdly, participation in activities is socially organized. This is a process that a novice becomes proficient for the use of cultural tools while participating in the cultural practice under the guidance of expert.

The third process is explained by the thought of Vygotsky's "psychological tool" (Vygotsky, 1984). The thought means that people do not react directly when they react a stimulus, but people intentionally create an artificial and auxiliary stimulus to react indirectly. As an example of psychological tool, languages, algebraic symbols, graphs, diagrams, and so on are followed. People can analyze problems and make future plans by using mediating stimulus that is not in the direct vision or territory of action. This paper interprets these functions of psychological tool as symbolizing. In other words, "a creation of a space in which the absent is made present and ready at hand" (Nemirovsky & Monk, 2000: 177). In this paper, we will adopt this definition.

In the cultural-historical theory, the process that people appropriate psychological tool is explained as follows. A tool (stimulus-object) exists outside, structures interpersonal connection, and then becomes individual psychological instrument. Vygotsky (1984) designed the settings where a person acquires his own stimulus-means by using a stimulus-object given by others. This paper stands in this point. Teacher provides pupils with stimulus-object that can be shared between teacher and pupils, then the teacher promotes so that the stimulus-object can be pupils' stimulus-mean.

DESIGN OF TEACHING UNIT

To consider proportion from the viewpoint of cultural-historical theory is to clarify (a) motive, (b) cultural tool, and (c) process that the teacher guides pupils.

Motive for Using Proportion

Proportion, generally, function is a mathematical way of knowing that is supported by the following motive. This is, "When there is a phenomenon we would like to control, but it is difficult for us to directly approach. If we can find related and approachable phenomenon, then we can control the more difficult one as well." (Shimada, 1990: 30). In this regard, Miwa (1974) suggests the following two points are essential. One is "projection", that is, observing a phenomenon from a different phenomenon makes the consideration of target easier. The other is "function", that is, considering what sort of characteristic and structure that the function conserves. This means to find regularity of correspondence and change, namely, to discover invariant or constant in quantity changing by the change of another value and to utilize them in problem solving.

Cultural Tools: Numerical Table, Graph, and Formula

When solving a realistic problem from the viewpoint of function, we systematically analyze data gained from experiment. The cultural tools to deal with the data are numerical table, formula, and graph. Numerical table is a tool regularly arraying the set of dependent variables when independent variables are changed systematically. If a table becomes a symbol, it is possible to acquire data that are not in the hand or to predict unknown values by using the table. A graph is a visual symbol that geometrically presents quantities that are not originally spatial or figural as a position or curve line (Sfard & Kieran, 2001). It visually represents the quantitative tendency, especially in the continuous quantity that comes into effect in the whole system. When the graph is made, the complicated relationship of proportion is demonstrated as a straight line, the simplest figure. When the graph is symbolized, it makes an effect to deduce the parameter by applying the relationship of proportion to the plotted data, and to reason or predict the phenomenon. The formula $y = a\,x$ compresses all the data and shows explicitly the way dependent variables are directly determined by the independent variables. This also fully comforts to the etymology of symbol "sum-ballen" that is "to combine". When a formula is dealt with as a symbol, an essential aspect underlying the problem situation can be recognized. In addition, we can determine that the phenomenon represents proportion judging from the manipulated formula, or describe the phenomenon based on the character of proportion.

Process that the Teacher Guides Pupils and Crucial Role of Discourse

In the early stage of unit, we use tables, graphs, and formulas as a social function between the teacher and pupils, namely as a notation for others. Tables, graphs, and formulas are not psychological tools of proportion. Actually, these are rather statistical than functional in quality, and are social means so as to record or present results for others. In the lesson, the teacher uses them as a social function and require pupils a higher theme. For example, in numerical table, pupil should search the data not from left to right, but see by jumping the space or interpolating the space. In a graph, pupil should not line the points and make a line graph, but line the space with understanding the all the points are lined in straight.

In the symbolization of notation, structuring discourse in the classroom by the teacher becomes more important. Firstly, description of character of proportion "when x-value becomes double, triple…, y-value also becomes double, triple…with the variation of x" is rather long and logically complicated in sixth graders. This also contains omitted expressions and terminologies. Therefore, the teacher must help pupils so that they can learn the officially used descriptions and expressions in mathematics and use them. Secondly, to understand the concept of function is to conceive phenomenon as a system. In other words, pupils must recognize the variation not as separate, but as a whole. However, the pupils' explanations are based on individual and concrete context which are only understood well by them. It means that the explanations lack generality. For this reason, the teacher must organize the discourse that explains an understandable and general system in the whole classroom. From the above viewpoint, we think that building up the foundation of social interaction between teacher and pupils, leading concrete meanings to generalization through discourse, and turning statistical expressions into functional symbols are key points for designing unit plan.

UNIT PLAN

We developed a teaching unit that consists of 5 subunit (A to E), 12 hours in total (Fig.1). The lessons have been conducted in the two classes in a public elementary school from the September 28th in 2000.

Subunit	Class Hours	Topics Covered
A	①②	Motivation
B	③④⑤⑥	Symbolizing table
C	⑦⑧⑨	Symbolizing graph
D	⑩⑪	Symbolizing formula.
E	⑫	Summarize

Fig.1 Teaching Unit Plan

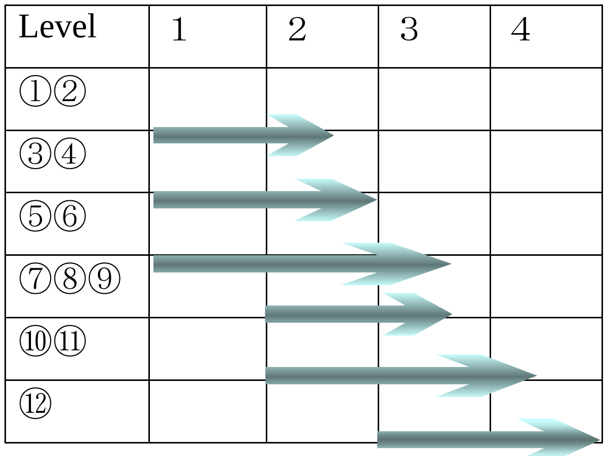


Fig.2 Attainment Levels in the Unit Plan

A feature of this teaching experiment is to adopt a motivation (projection and function) that supports an idea of function throughout the unit. When pupils think about a table, "sideways relation" tends to be strong and "longitudinal relation" tends to be weak. This teaching experiment researches a possibility to reduce the pupils' tendency to avoid the use of external ratio by keeping a motivation of projection.

The structure of the units consists of 4 attainment levels in accordance with RME. The first two levels are same as the RME's, but in the level 3, notation as a social function gradually turns into a symbol as a thinking function. Regarding this, we design a teaching plan so that a table at the second subunit (B), a graph at the third subunit (C), and a formula at the fourth subunit (D) can turn into a symbol. Especially, the second stage is not only a symbolization of a table, but a base of symbolization of a graph and formula in the third and fourth subunit, and we expect to induce a development of "meta representational knowledge" (Gravemeijer et al., 2000: 233) with regard to each feature and difference. In the level 4, we expect that pupils to utilize a table, a graph, and a formula as a symbol. More specifically, we expect that the pupils detect the structure of proportion from a subtle character in a concrete situation and apply it, or assume proportion and solve the problem. Level structure of the unit is shown in Fig.2.

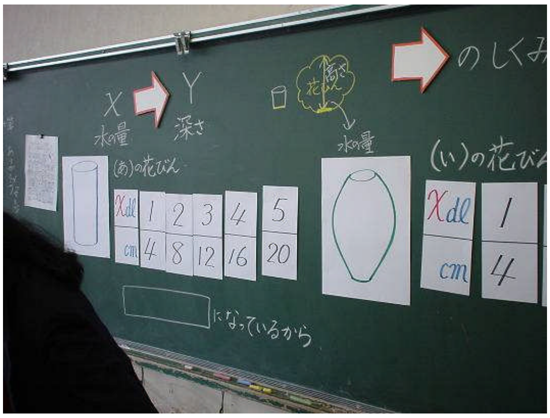
SYMBOLIZATION AND THE ROLE OF DISCOURSE

We will discuss the process that notations and expressions become a symbol with regard to discourse. We use here a table as an example. It is because that a table itself does not represent properties of proportion comparing to a graph or a formula. Therefore, to expose the property of proportion, we have to use language and pictorial arrow expression. We think there are two major roles of discourse about a table. One is to conceiving data sets as a system and the other is for linguistic formulation. We will discuss the former one below. Conceiving data set as a system needs an explanation not about a specified pair of values in a table, but in a general structure in whole the table.

Sort the data in Statistical table

In general, we often deal with a table in which data are sorted from the beginning in the study of proportion. At the third hour of class (③), the depth of water (*y* cm) is asked as a problem when water (*x* dl) was poured into the (a) cylindrical-shaped vase and (b) pot-shaped vase with a cup and the data were given randomly. The question "Is the condition of water different" worked as a trigger for pupils, they begun to sort the data to recognize easily and tried to find the tendency (Pic.1).

As a result, they concluded that "(a) may have a rule". Then the teacher asked *y*-value when $x = 8$ by using the table in which 1 to 5 of *x*-values were given. This question indirectly required to consider the solution by applying a rule of given table. 13 out of 35 pupils immediately raised their hand, but later almost all the pupils could say right answer. This result implies that for pupils the table was a matter that consists of given data at first, but later it became a tool to consider unknown values based on the rule.



Pic.1 Sorted Data

Explain calculation procedure with fragmentary rule

The pupils said together "32 cm" to the question of the unknown data of 8 dl. Most of explanation was incomplete even if the rule appeared or disappeared in their description. For example, a pupil said that the depth would be a multiple of 4 and the *x*-value would be 8, but he did not say the relationship between *x* and *y*. Another explanation was "sum would correspond to sum", but the pupils did not seem to understand. Also the explanations of external ratio and of internal ratio were made. Thus, the calculation procedure of pupils was brought to the fore to acquire the answer 32 by using an individual number, but they did not explain the general rule but replied only the calculation procedure or fragmentary rule using a specified pair of values and lacked generality.

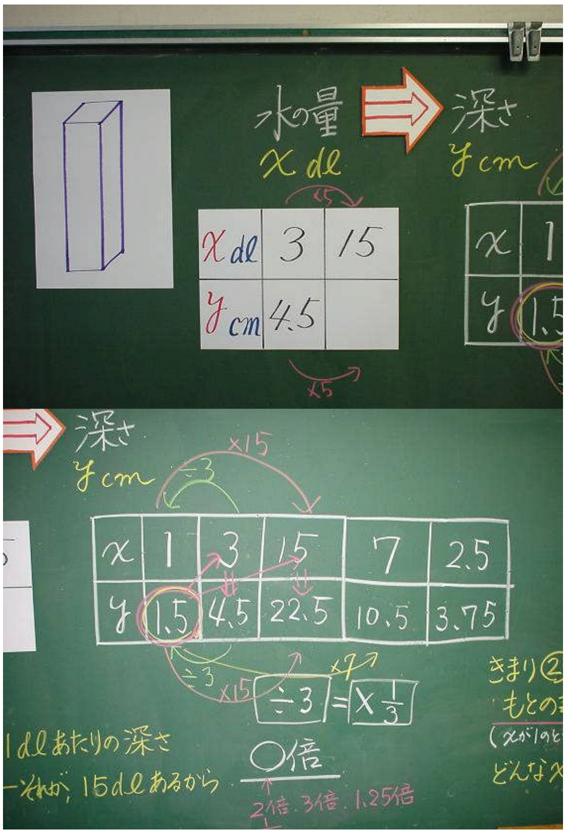
Pic.2 Semi-general Rule

When we exposed the general rule for the explanation based on the concrete relationship between values, the teacher considered that the explanation by word would be difficult, so he required pupils to show their thought on the table with an arrow sign. The arrow sign represented the rule of table and became an important tool in order to target the rule. The pupils were gradually detecting simple semi-general rule with teachers guidance. By “semi-general”, we mean the rule was based on “a number per 1” (unitary method). At the moment, the following property: “when *x*-value becomes double, triple…, *y*-value also becomes double, triple…with the variation of *x*” meant all the allow started from “a number per 1”(Pic. 2).

Detect general relations in the table

At the fourth hour, the teacher provided a higher level question than acquiring a *y* value from a pair of data. The problem was, “when a value is 3 dl, the other value is 4.5 cm, then when a value is 15 dl, what cm is the other value? (Pic.3 above)

“A number per 1” (unitary method) came up in the discourse when we focused on this solution (Pic.3 below). Some pupils elaborated “zigzag” method which was transitional one and mixture of inner and external ratio. The meaning of “a number per 1” for pupils was the *y*-value when *x* = 1, but for the teacher, the value was a proportional constant. The fact that same wording has may meanings constitutes so called “Zone of Proximal Development” (Vygotsikii, 1984) in social interaction. The teacher made the term “a number per 1” for a target of discourse, and the pupils considered where the number can be seen in the table. The constant value begun to work as a symbol of proportion when “a number per 1” could be seen in whole the table. Thus, the teacher established a base of interaction with pupils while showing a higher level of problem and designed discourse so that the pupils could pay attention to the general rule behind the table.



Pic.3 Problem and Solutions

Symbolized table become operational

Pupils also found the defining character of proportion: conservation of sum. At the sixth hour, the teacher posed more difficult problem. The problem was, “when a value is 2.5dl, the other value is 3cm, and when a value is 6.5dl, the other value is 7.8cm, then when a value is 9dl, what cm is the other value? When a table became a symbol, it was possible to detect properties here and there, and acquired data that are not in the hand (Pic. 4).

Pic. 4 Symbolized Table

CONCLUDING REMAKS

We proposed a unit design for teaching proportion in a sixth-grade of Japanese elementary mathematics classroom based on cultural-historical theory in which cultural tools such as table, graph and formula become symbol of proportion in collective discourse. In this report, we described a hypothetical learning teaching trajectory only for numerical table. Results from the interpretation of the data reveal that the process of symbolizing consists of four phases. This hypothetical trajectory could be applied for of symbolizing graph and formula as well. Teaching experiment reveal that classroom collective discourse functions as social resources for promoting process of symbolization. Through collective discourse, the cultural tools are gradually appropriated by the pupils as cognitive means for regulating their personal mathematical activity. Thus, process of symbolizing of cultural tools is characterized by changes of their function form collective use to private one.

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