BRIDGES AND OBSTACLES: THE USE OF LESSON STUDY TO IDENTIFY FACTORS THAT ENCOURAGE OR DISCOURAGE MATHEMATICAL THINKING AMONGST PRIMARY SCHOOL STUDENTS

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The first part of the paper describes a study conducted to explore the use of lesson study as a professional development tool. The specific aims of the study and the main stages of the study are described. The second part describes how lesson study helped teachers understand the factors that encourage or discourage mathematical thinking during a lesson. The third part briefly discusses the role of lesson study in enhancing teachers' pedagogical knowledge. The final part outlines a research agenda that employs lesson study to help teachers develop approaches to cultivate mathematical thinking amongst students.

INTRODUCTION

The Singapore mathematics curriculum which focuses on mathematical problem solving was introduced in 1992 and was revised in 2001 and, again, in 2007. Increasingly, the shift with each revision of the curriculum is less emphasis on computational, procedural skills and more emphasis on mathematical thinking. Mathematical thinking is integral in the process of problem solving.

It is, thus, important for teachers to understand the idea of mathematical thinking and how to cultivate it amongst students. However, teachers need to re-examine their own mathematical thinking and their perception of what mathematical thinking is.

Lesson study provides a concrete image and specific situations of mathematical thinking amongst students as they unfold in a classroom. The research lessons provide opportunities to capture the complexities in understanding what mathematical thinking is and the pedagogy associated with its development, that otherwise may not be captured.

THE STUDY

A group of eight teachers in a primary school in Singapore was involved in a six-week lesson study cycle. The aim of the study was to explore the use of lesson study as a professional development tool. In particular, the study reported in this paper focused on two goals. One goal was to enhance the teachers' pedagogy with respect to cultivating mathematical thinking. The other goal was to enhance the teachers' own mathematical thinking and their understanding of mathematical thinking.

In the first session, the teachers were familiarized with the ideas of visualization and generalization as possible aspects of mathematical thinking (Yeap, 2006). The teachers

then used a topic (angles) that they were going to teach in the coming weeks to anchor their discussion. The teachers studied the textbooks, workbooks, teachers' guides and other resources that were available including manipulative materials. The discussion culminated in the teachers identifying ideas in the topic of angles that would be a challenge or otherwise for the primary four (grade four) students. The research theme for the research lesson was decided to be helping student construct a visual representation of angles with a focus on a representation that was thought to be challenging for the students. It was thought that students find it difficult to form a visual representation of an unknown angle a when $a + b$ is known (Figure 1).

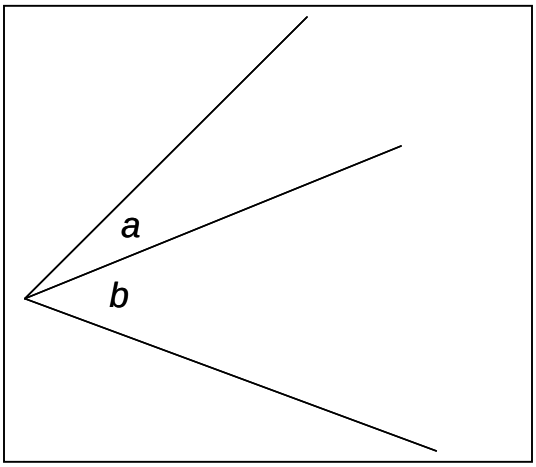

Figure 1: While students are able to tell the angle $a + b$ when angles a and b are given, they find it more challenging to tell an unknown angle when $a + b$ and either a or b are given.

In the second session, the teachers designed the lesson, wrote the lesson plan and made the necessary preparation for the research lesson. They worked out the solutions of the problems they planned to pose to the students and the anticipated students' responses.

As the focus of the lesson study was to develop a possible approach to help students construct a visual representation of angles with a particular emphasis on representations that are considered challenging to the students, the teachers decided to employ the use of concrete materials. Cut-outs of sectors that show 20^o or 30^o were prepared. The use of these cut-outs was, in the opinion of the teachers, helpful in assisting students construct the target mental representations.

The teachers started with the problem of showing different angles using the cut-out pieces. As they solved this problem, they found that the problem was too open. They had to decide on the number of each type of cut-outs to use. They had to decide on how the cut-outs were to be arranged – adjacent to each other, on top of each other or a combination of the two methods. For example, a 20^o piece and a 30^o piece can be placed side by side to show an angle of 50^o. A 20^o piece can also be placed on top of a 30^o piece

to show an angle of 10^o. As they solved the problem, they also predicted that students would find the latter challenging, although the use of cut-outs may be useful.

According to the lesson plan, students were to be given a certain number of cut-outs of sectors that show 20^o or 30^o. In this lesson, students were asked to use (a) a 20^o piece and a 30^o piece, and (b) two 20^o pieces and a 30^o piece. Students were required to use these to show other angles.

The research lesson and a post-lesson discussion were conducted in the third session. One of the teachers taught the lesson to a primary four class. Each teacher observed one group of students. The teachers were reminded that they needed to observe students carefully to collect information on student thinking. The next part of this paper focuses on how this research lesson and the information collected helped the teachers identify factors that encouraged or discouraged mathematical thinking during a lesson.

The fourth session focused on revising the lesson plan based on the findings of the research lesson. Subsequently, another teacher taught the lesson to a different primary four class. A post-lesson discussion was again conducted. The final session was spent identifying parts of the lesson plan where mathematical thinking is prominent and delineating teacher actions that are able to stimulate, scaffold, encourage and perpetuate mathematical thinking.

FACTORS THAT ENCOURAGED OR DISCOURAGED MATHEMATICAL THINKING

The first research lesson was made of five main segments. Figure 2 describes the structure of the first research lesson which was 30 minutes long.

Segment	Description	Time
1	The teacher reviewed the idea of angles generally.	00 : 00
2	The teacher helped students understand the task for the lesson using one red (20^o) cut-out and one green (30^o) cut-out.	03 : 10
3	The students worked in groups using two green (30^o) cut-outs and one red (20^o) cut-out.	06 : 55
4	The teacher used two groups' solutions to lead a whole-class discussion.	12 : 00
5	The teacher conducted a general conclusion to the lesson	25 : 50

Figure 2: The structure of the first research lesson

The post-lesson discussion focused on the research theme – to develop a possible approach to help students construct a visual representation of angles with a particular

emphasis on representations that are considered challenging to the students. The bulk of the post-lesson discussion was on the factors that encouraged or discouraged mathematical thinking. The following paragraphs are a synthesis of the post-lesson discussion.

The use of the cut-outs was critical in helping some students construct a visual representation of the central ideas of the lesson. This was particularly true in the challenging cases.

Many students did not face difficulty when the cut-outs were placed adjacent to each other. Thus, many students were able to see how a green piece and a red piece could show 50^o readily. The role of the cut-outs differed among different students in this situation. There were students to whom the cut-outs did not matter. They could say how 50^o could be shown without using the cut-outs. These students already had the visual representation of the idea and were making use of it to complete the tasks confidently. Then, there were students who used the cut-outs to strengthen their visual representation. They could say how 50^o could be shown but used the cut-outs to confirm their thinking. Finally, there were students who needed to use the cut-outs to arrive at the conclusion of how 50^o could be shown.

Many students had difficulties when the cut-outs were placed on each other. Thus, not many students were able to see how a green piece and a red piece could show 10^o by placing the red piece on top of the green piece in a certain way. The few students who could still needed the cut-outs to confirm their thinking.

The majority of the students needed the scaffolding provided by the teacher to make the cut-out useful in developing a visual representation of the idea. As the scaffolding was important, the teachers agreed that they needed to be more rigorous in developing the scaffolding questions. This was done for the second research lesson and the positive effects of carefully-constructed scaffolding questions were apparent.

The arrangement for students to work together in groups provided opportunities for students to encounter responses that differed from one's own. This led to students questioning their peers, seeking clarifications, defending their responses and resolving conflicting views. Such extended engagement with ideas was found to be conducive for mathematical thinking.

The use of the worksheet did not allow for such extended engagement. Answers had to be obtained and recorded promptly. In completing such a worksheet, the students were more eager to have an answer they can record to the teachers' satisfaction. There was little opportunity for engagement with ideas. It was decided that it would be better not to require students to complete a worksheet where answers had to be obtained and recorded quickly. In the second research lesson, the worksheet was not used. Instead, students

were given an individual worksheet at the end of the lesson to consolidate the ideas that they had discussed in the lesson.

While the majority of the lesson was focused on a set of related problems, the first and last segments of the lesson were too general to be useful. General, superficial discussion of ideas did not facilitate mathematical thinking. On the other hand, students working on one problem that a set of solution ranging from obvious ones to challenging ones facilitated mathematical thinking. In the second research lesson, these segments were removed without affecting the main aims of the lesson. The time was used instead to complete the individual worksheet at the end of the lesson.

The problem used in the lesson was open enough to engage students in mathematical thinking. However, the teacher provided the suggestion that the pieces could be placed on each other even before the students had a chance to consider it. This premature direction robbed the students with a chance to make sense of the situation. In the first research lesson, there were students who simply placed the red piece on the green pieces without understanding its significance. This was because the teacher had said that the pieces could overlap. This suggestion was not given in the second research lesson. While fewer groups came up with this method of showing angles independently, these groups need no further help from the teacher in understanding its significance.

The information the teachers collected during the research lesson had resulted in teacher understanding of factors that facilitated mathematical thinking and those that were obstacles to mathematical thinking. Generally, the following was found to be a bridge to mathematical thinking: (a) the use of concrete material to anchor students' thinking, (b) the use of carefully crafted scaffolding questions to help student clear challenging situations a step at a time, and (c) extended engagement with ideas where students encountered different and, sometimes, conflicting views and where they had to question, clarify, justify and defend ideas. The following were found to be obstacles to mathematical thinking: (a) the use of worksheet that required a response to be recorded promptly, (b) the use of closed problems or the conversion of open problems to closed ones by providing directions too early in the problem-solving process.

LESSON STUDY IN DEVELOPING PEDAGOGICAL KNOWLEDGE

The data collected from this study involving eight teachers going through one lesson study cycle in helping teachers develop approaches to cultivate mathematical thinking amongst students allows a brief discussion on the use of lesson study in developing pedagogical knowledge.

In the lesson planning phase, solving the problems themselves allowed teachers to experience mathematical thinking and clarify to themselves what mathematical thinking means. In solving the problem in this study (finding angles that can be shown using a number of cut-outs that show 20^o and 30^o), one teacher very quickly realized the idea that

"all multiples of twenty and thirty can be shown", to which another teacher extended when she said "so can all multiples of fifty". The former later included generalizing as an important part of mathematical thinking. Another teacher saw that 20^o can be shown by placing one cut-out on the other. He was made to clarify what he meant and to justify his thinking as several of his colleagues did not understand him. He included defending one's idea as an important part of mathematical thinking. In lesson study, the lesson planning stage included opportunities to reflect and articulate one's thinking in solving the problems selected for the lesson. In individual lesson planning, the reflection and articulation opportunities are left to chance.

In the research lesson phase, observing the students' thinking closely allowed teachers to see mathematical thinking in action. They are also able to see aspects of mathematical thinking that are easy for the students and those which are challenging. Teachers are also able to see instructional strategies that facilitate or inhibit mathematical thinking. In cases where the teachers have the opportunity to revise the lesson plan and conduct a second research lesson, they are able to test their conjectures. The research lesson also shows up instructional strategies that require more careful planning. In this study, the teachers initially did not realize the need to plan the scaffolding questions closely. As a result, the challenging part of the problem (the case of overlap) was not grasped by many students. In the revised lesson plan, the scaffolding questions were carefully crafted. This revised action bore positive effects in the second research lesson. The research lessons, thus, have the twin roles of showing the facilitating or inhibiting effects of instructional strategies including when these strategies are absent or not rigorously designed.

A RESEARCH AGENDA

In Singapore, professional development courses offered by the National Institute of Education are typically in the form of 24-hour courses. A new in-service course in the form of lesson study will be proposed. The structure of the course will be similar to the one described here with an initial session to introduce the lesson study process and a final session to allow teams to share their experience.

The research questions are (1) How do teachers develop their pedagogy in cultivating mathematical thinking amongst primary school students through lesson study? (2) What are the effects of lesson study on the teachers' mathematical thinking, perception of what mathematical thinking is and pedagogical knowledge of cultivating mathematical thinking?

Instruments will be developed to collect data for teachers' mathematical thinking and their perception of what mathematical thinking is. Changes in teachers' pedagogical knowledge will be based on field notes collected during the sessions to study the instructional materials, to plan and revise lesson, to discuss the research lessons and to identify specific points during a lesson where there is significant mathematical thinking and instructional strategies that support it.

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