TEACHERS' MATHEMATICAL VALUES FOR DEVELOPING MATHEMATICAL THINKING TROUGH LESSON STUDY

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1. Mathematical thinking from a sociocultural perspective

Mathematical thinking sounds like an essentially psychological topic. It is just another branch of thinking, and therefore part of the psychological field of knowledge. However, we can never observe mathematical thinking - we can only observe what we assume to be its products, namely mathematical ideas and processes. We can also observe what conditions and contexts might have been responsible for the products of mathematical thinking, which brings us rather closer to the social context.

So what is the problem we are trying to consider here? In one sentence the major problem seems to be: "How can teachers help mathematical thinking to develop in their students?" A subsidiary problem is "How can research on values help with this?" Because of my research work in the field of mathematics education, I prefer to consider mathematical thinking not from a psychological perspective but from a socio-cultural perspective.

2. Three theoretical ideas

In trying to make research progress in solving the problem of helping mathematical thinking develop, I believe we need to consider carefully any theoretical perspectives which might assist us. I will present here three theoretical ideas which I have found helpful in my research and which I believe can shape our understanding of the problem and lead to potential pedagogical solutions. These 'solutions' can then be researched using the Lesson Study method – but more about that later.

2.1 Lancy's developmental theory of cognition

David Lancy (1983) is a cultural psychologist who, in his major cross-cultural study in Papua New Guinea, developed a new stage theory of cognition. It was based on Piaget's theories but he developed them from a socio-cultural perspective. He was doing his research in Papua New Guinea and through investigating cognition with students in PNG, he found that the theoretical developmental sequence of Piaget's stages were similar to, but not identical with, those Piaget found in his European-based research.

He found that Stage 1 was very similar to Piaget's sensory-motor and early concrete operational stages. He argued that this stage is where genetic programming has its major influence, and where socialisation is the key focus of communication. Many activities involving the child are completely similar across cultures.

He then argued that Stage 2, a later concrete operational stage, is where enculturation takes over from socialisation. As he says: “Stage 2 has much to do with culture and environment and less to do with genetics”, and he demonstrated that this is the stage where different cultures will emphasise different knowledge and ideas. Even in relation to mathematics (which is where ethnomathematics develops) this is the case..

The big development in Lancy’s theory from Piaget’s is seen in Stage 3 which concerns the meta-cognitive level. Lancy says: “In addition to developing cognitive and linguistic strategies, individuals acquire ‘theories’ of language and cognition.” Different cultural groups emphasise different ‘theories of knowledge’ and Piaget’s ‘formal operational’ stage is one such theory of knowledge emphasised in Western culture. Confucian Heritage Cultures emphasise other theories of knowledge. These theories of knowledge represent the ideals and values lying behind the actual language or symbols developed by a cultural group.

Thus it is in Stages 2 and 3 that values are inculcated in the individual learners. In a classic work by Kroebner and Kluckholm (1952) they strongly support this idea: “Values provide the only basis for the fully intellible comprehension of culture, because the actual organisation of all cultures is primarily in terms of their values” (p. 340).

Thus for our problem, the idea of mathematical thinking as a form of meta-cognition, affected by the cultural norms and values of the learner’s society, is helpful.

2.2 Billett’s (1998) analysis of the social genesis of knowledge.

But where do these norms, values and knowledge come from, and how can we think about them from a more educational perspective? Stephen Billett’s (1998) sociological work analyses and locates what he calls “the social genesis of knowledge” in 5 inter-relating levels:

Socio-historic knowledge factors affect the values underpinning decisions made by both institutions and teachers. It is knowledge coming from the history and culture of the society, and is value-laden knowledge.

Socio-cultural practice is defined by Billett as historically derived knowledge transformed by cultural needs, together with goals, techniques, and norms to guide practice. At the institutional level these are manifested by curricular decisions influenced by such factors as: (a) current institutional management philosophy with respect to educational and social values (*in loco parentis*); (b) State or national curricular frameworks and (c) the ethos of the mathematics faculty or teacher’s peer group.

The community of practice in the classroom is identified by Billett as particular socio-cultural practices shaped by a complex of circumstantial social factors (activity systems), and the norms and values which embody them. This community is influenced by (a) the teachers' goals with respect to and portrayal of pedagogical values, (b) students' goals and portrayal of learning values, and personal values.

Microgenetic development is interpreted by Billett as individuals' (teachers' and students') moment-by-moment construction of socially derived knowledge, derived through routine and non-routine problem solving. The nature of teaching as a profession is reflected in the *relative* autonomy assumed within the walls of the classroom, where teachers' decisions are constantly being made or revised on the basis of a continuous flow of new information. The instantaneous nature of many decisions is likely to be influenced to a greater or lesser extent by the teacher's internalised sets of values.

Ontogenetic development includes individuals' personal life histories, socially determined, which furnish the knowledge with which to interpret stimuli; this development includes participation in multiple overlapping communities.

This analysis points to the different sources of influence on teachers' values. Billett's categorised knowledge is a powerful indicator of how different knowledge at these five levels can impinge on and influence teachers' values in the classroom.

2.3 Bishop's (1988) socio-cultural dimension and its levels

My research context has been in the field of culture, and especially with considering mathematics as a form of cultural knowledge. When we are considering how to develop values in relation to mathematical thinking, I also believe we need to keep in mind the socio-cultural dimension of mathematics education. This dimension influences the values of mathematical thinking at five levels, which are similar but different to Billett's levels.

1. Cultural level – the overarching culture of the people, their language, their mathematics, their core values. In Billett's levels he combined together the cultural and the societal, which I believe in the case of mathematics education is not helpful. Evidence from research at the cultural level shows how different ethnomathematical ideas are not necessarily related to similar societal structures. Ethnomathematics points to cultural influences on mathematical thinking.
2. Societal level – the social institutions of the society, their goals, and their values regarding mathematics. In many societies mathematics education is a contested field with many proponents of different educational 'solutions' vying for publicity and academic advantage. They inevitably affect what is considered to be important mathematical thinking, and who is capable of doing it.
3. Institutional level – the educational institutions' values and the place of mathematics within them. At this level we can see the ways institutional values influence the curriculum, the timetable and even the allocation of space to each subject. These values also affect the development of mathematical thinking in different groups of students.

4. Pedagogical level – the teachers' values and decisions, the classroom culture of mathematical thinking. This is the same level as Billett's 'community of practice', and I have to confess that I prefer Billett's description of this level, as it emphasises the contribution of teacher and students to the classroom knowledge culture.
5. Individual level – individual learners' values and goals regarding mathematics, and mathematical thinking, which can differ markedly, and which do not necessarily follow the teachers' values and goals.

Thus I will draw on these three perspectives in the rest of this talk, and in particular I will assume that my ideas about values regarding mathematical thinking are:

1. Concerned with developing metacognition
2. Located within the socio-cultural dimension
3. Focused on the community of practice in the classroom.

3. Values and mathematical thinking

Now we turn to the values problem stated in Section 1 above. Building on the above analysis, I realised firstly that it was necessary to distinguish between three kinds of values:

- Mathematical values: values which have developed as the knowledge of mathematics has developed within any particular culture.
- General educational values: values associated with the norms of the particular society, and of the particular educational institution.
- Mathematics educational values: values embedded in the curriculum, textbooks, classroom practices, etc. as a result of the other sets of values.

My research approach to values and mathematical thinking has been to focus on mathematical values, and on the actions and choices concerning them (see Bishop, 1988, 1991, 1999). In my work I have used White's (1959) three component analysis of culture:

- Ideological component: composed of beliefs, dependent on symbols, philosophies,
- Sentimental (attitudinal) component: attitudes, feelings concerning people, behaviour,
- Sociological component: the customs, institutions, rules and patterns of interpersonal behaviour.

So how are these components interpretable in terms of mathematical thinking?

3.1 The Ideological component of Mathematical values

In regards to this component of the Mathematical culture, I argued (Bishop, 1988, 1991) that the critical values concern *Rationalism* and *Objectism*.
Valuing Rationalism means emphasising argument, reasoning, logical analysis, and explanations, arguably the most relevant value in mathematics education.

Ask yourself as a teacher:
Do you encourage your students to argue in your classes?
Do you have debates?
Do you emphasise mathematical proving?
Do you show the students examples of proofs from history (for example, different proofs of Pythagoras' theorem)?

Valuing Objectism means emphasising objectifying, concretising, symbolising, and applying the ideas of mathematics.

Ask yourself:
Do you encourage your students to invent their own symbols and terminology before showing them the 'official' ones?
Do you use geometric diagrams to illustrate algebraic relationships?
Do you show them different numerals used by different cultural groups in history?
Do you discuss the need for simplicity and conciseness in choosing symbols?

3.2 The Sentimental (Attitudinal) component of Mathematical values

In regards to this component, the important values are *Control* and *Progress*.

Valuing Control means emphasising the power of mathematical and scientific knowledge through mastery of rules, facts, procedures and established criteria.

Ask yourself:
Do you emphasise not just 'right' answers, but also the checking of answers, and the reasons for other answers not being 'right'?
Do you encourage the analysis and understanding of why routine calculations and algorithms 'work'?
Do you always show examples of how the mathematical ideas you are teaching are used in society

Valuing Progress means emphasising the ways that mathematical and scientific ideas grow and develop, through alternative theories, development of new methods and the questioning of existing ideas.

Ask yourself:
Do you emphasise alternative, and non-routine, solution strategies together with their reasons?
Do you encourage students to extend and generalise ideas from particular examples?
Do you stimulate them with stories of mathematical developments in history?

3.3 The Sociological component of Mathematical values

In regards to this component, the important values are *Openness* and *Mystery*.
Valuing Openness means emphasising the democratisation of knowledge, through demonstrations, proofs and individual explanations.
Ask yourself:
Do you encourage your students to defend and justify their answers publicly to the class?
Do you encourage the creation of posters so that the students can display their ideas?
Do you help them create student math newsletters, or web-pages, where they can present their ideas?

Valuing Mystery means emphasising the wonder, fascination, and mystique of mathematical ideas.
Ask yourself:
Do you tell them any stories about mathematical puzzles in the past, about for example the 'search' for negative numbers, or for zero?
Do you stimulate their mathematical imagination with pictures, artworks, images of infinity etc.?

These then are what I believe to be the crucial values underpinning the development of mathematical thinking in the classroom. I think we will make good progress in solving our problems if more research is devoted to investigating ways of developing these values in our teachers, so that they can develop them in their students.

4. Values, Mathematical Thinking and Lesson Study

Researching values development is no easy matter, but Lesson Study is an excellent method for studying the development of values in the classroom. In our Values and Mathematics Project (VAMP) we already used a version of lesson study, but without trying to affect the teachers' plans for their lessons.

1. The teachers told us before the lessons what values they thought they were going to develop.
2. We observed and recorded the lessons
3. We interviewed the teachers after the lessons to have them explain what they thought they had achieved.

More details of this research can be found at:
http://www.education.monash.edu.au/research/groups/smte/projects/vamp/vamppublications.html

For a full lesson study of mathematical thinking values, it would be necessary to plan together with the teachers what values they would try to develop.
The teaching ideas earlier would be very appropriate for this. It would be important for the experiment to go over a group of lessons, as values could hardly be developed in one lesson.

5. Conclusions for research

1. With any design and development research it is essential to have good theories to support and structure the work
2. Mathematical thinking has been studied in many ways, but in relation to values it is important to consider it as an aspect of meta-cognition.
3. The context for the research should be the classroom, as it is there that the community of practice significantly influences the meta-cognitive aspects of mathematical thinking.
4. Equally important to consider in this research is the socio-cultural context, as any educational values are embedded in the culture of the society.
5. Lesson study is an excellent research approach for studying any experimental educational development.
6. It is particularly appropriate for studying values development.
7. However there need to be a series of lessons studied as values do not develop in the space of one lesson.
8. Finally the teachers need special support in this research, as values teaching involve the teacher's pedagogical identity, which must be respected (Chin, Leu & Lin, 2001).

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